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Quasi-Gaussian point source function and two approximations of its out-of-focus intensity distribution

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Introduction

There are many cases in the applied optics when the point-spread function of the desired optical system is not diffraction limited (Airy's pattern) but seems like a blur spot with a size much more greater than the diameter of the central Airy's disk. The most trivial examples of this kind are stellar (i. e., point sources) turbulent images observed by means of ground-based large optical telescopes. If the optical system is not precisely focused, the resulting point source intensity distribution $g(r)$ will differ from that in the focal plane (x_0, y_0) depending on the distance Δf between the plane (x_0, y_0) and the plane (x, y), where the intensity is measured by the detectors (Fig. 1).

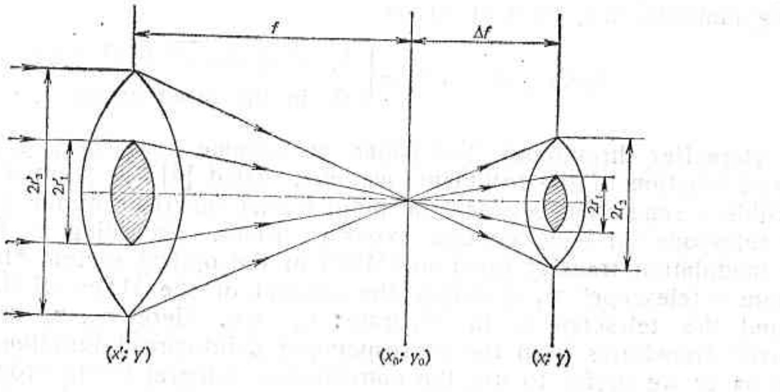

Fig. 1. Scheme of the optical system
(x', y') — input aperture plane; (x_0, y_0) — focal plane; (x, y) — out-of-focus plane;
f — focal length; Δf — out-of-focus distance; $\varepsilon = r'_1/r'_2 = r_1/r_2$ — central screening

We shall also denote by prime's the quantities referred to the input aperture plane (x', y') and by f the focal length. For concreteness, it is considered the case where the plane (x, y) is behind the plane (x_0, y_0) $(\Delta f > 0)$. Further, we shall assume that the circular input aperture of the optical system has a central screening (shaded region in Fig. 1), as it is usual for the large telescopes. According to the accepted in this paper geometrical optics approach, the illuminated area is a ring with inner radius r_1 and outer radius r_2 $(r_2'/f = r_2/\Delta f$; Fig. 1). The screened part of the input (output) aperture is characterized by the parameter $\varepsilon = r_1'/r_2' = r_1/r_2 = \text{const} < 1$.

During the last two decades many new methods for restoration of distorted images are developed and their practical realization is proven to be useful [1, 2, 3]. In particular, such techniques are applied in optical astronomy for development of out-of-focus images (Hubble Space Telescope) and images obtained through the turbulent Earth's atmosphere. The later case is implicitly considered as a preferred range of applicability of the results obtained in this paper.

Let us denote by $G_0(x_0, y_0)$ the intensity distribution of a precisely focused image and by $G(x, y)$ the intensity distribution of the same image in the out-of-focus plane (x, y) where the light sensitive detectors (photographic emulsion, CCD-matrix, etc.) are placed. Supposing that the principle of the linear superposition is fulfilled, the relation between these distributions is given (in the absence of noise) by

$$G(x, y) = \iint_{-\infty}^{\infty} G_0(x-\xi, y-\eta) h(\xi, \eta) d\xi d\eta, \tag{1}$$

where the integration over the coordinates ξ and η is performed in the out-of-focus plane. In practice, if the source is very bright, we may integrate over the area where the ratio signal to noise is greater than unity. In the above expression $h(x, y)$ is the point-spread function of the considered out-of-focus optical system, e. g., this is the intensity distribution in the out-of-focus plane (x, y) when the system is illuminated by light rays parallel to the principal axis. According to the accepted geometrical optics approach, within a normalizing multiple, $h(x, y)$ is given by

$$h(r = \sqrt{x^2 + y^2}) = \begin{cases} 1, & \text{if } r_1 \le \sqrt{x^2 + y^2} \le r_2; \\ 0 & \text{in the other cases.} \end{cases} \tag{2}$$

Hereafter throughout this paper we assume that $G_0(x_0, y_0)$ is the point-spread function of the tutbulent medium. Fried [4] has pointed out that it is possible a separation between the point-spread functions of the atmosphere and the telescope for long enough exposure times. According to Fried's results, the modulation transfer function (MTF) of the optical system "turbulent atmosphere + telescope" τ_{at} is simply the product of the MTFs of the atmosphere τ_a and the telescope τ_t in separate: $\tau_{at} = \tau_a \tau_t$. Here τ_{at}, τ_a and τ_t are the Fourier transforms from the corresponding point-spread functions. However, in this paper we prefer to use the convolution integral (1) in order to compute the long-exposure point-spread function $g(r)$ of the combination "turbulent medium + optical device" system (without including the detector responce). Concretely, we shall investigate the distortions caused by the out-of-focus registration of the intensity distribution conditioned by point source, observed through a turbulent medium. To specify this case (as we have already done above), we use small letters for the point source intensity distributions $g_0(x_0, y_0)$ and $g(x, y)$ (i. e., point-spread functions) in the focal plane (x_0, y_0) and in the out-of-focus plann (x, y), respectively.

Approximation by a quasi-Gaussian function. Qualitative treatment of the problem

In this paragraph we shall assume that every of the observed point sources causes a quasi-Gaussian intensity distribution $g_0(r_0)$ of the light in the focal plane (x_0, y_0) of the optical system

(3) $$g_0(r_0=\sqrt{x_0^2+y_0^2})=S_0 \exp(-r_0^{2n_0}/B_0),$$

where $B_0=(2\sigma_0^2)^{n_0}$ is a constant determining the size of the circular spot. $S_0=S_0(\sigma_0)$ is a normalization constant depending on the total energy flux of the image and its numerical evaluation is not important in this paper, because we are interesting only on the relative intensity distributions within the images. The power n_0 describes the deviation of the distribution $g_0(r_0)$ from the Gaussian one $(n_0=1)$. We assume that $n_0=$constant for the whole area of the image (i. e., n_0 does not depend on r_0). We emphasize that the later statement is true for the focal plane (x_0, y_0) and, generally speaking, is not true in the out-of-focus plane (x, y), where the intensity is measured by the detectors.

The observed intensity distribution $g(x, y)$ may also be fitted by a quasi-Gaussian function

(4) $$g(r=\sqrt{x^2+y^2})=g(x, y=0)=S\exp[-x^{2n(x)}/B]; \quad B=(2\sigma^2)^{n(x)},$$

but, generally speaking, we expect that for this approximation the power is not a constant and will depend on r (or x, because we investigate the intensity distribution in the direction $y=0$). Obviously, the point source out-of-focus images have lower central intensities $(S<S_0)$ and are enlarged $(B>B_0)$. If $|\Delta f|$ is greater, the redistribution of the light energy from the central part of the image to its outer part is a more pronounced effect (but the total light flux is not changed). If we assume that the power $n(x)$ does not vary too fast into the interval $(x-\Delta x, x+\Delta x)$, where $\Delta x \ll \sigma$ (i. e., we consider $n(x)$ locally as a constant), we can obtain the following expression [5]

(5) $$n(x)=0{,}5\left[1+x\left(\frac{g''(x)}{g'(x)}-\frac{g'(x)}{g(x)}\right)\right],$$

where the prime's denote differentiating with respect to x. By means of the above approximate expression we are able to evaluate the global (with respect to the size of the image) changes of the power n which describes the slope of the intensity distribution $g(x)$.

In this section we shall assume that the power n_0 in the expression (3) has a constant value for the whole area of the (precize focused) image, but its values, generally speaking, are not equal to unity. According to this acceptance, we shall also consider the powers $n_{04}>n_{03}>1>n_{02}>n_{01}$ which do not depend on x. The relative comparison between distributions $g_0(x; n_{01})$, $g_0(x; n_{02})$, $g_0(x; n_{03})$ and $g_0(x; n_{04})$ is given in Table 1. It would be pointed out that

(6) $$g_0(0; n_{0i})=S_0=1; \quad (i=1, \ldots, 4)$$

Table 1

Comparison between quasi-Gaussian intensity distributions for different values of the power n_0

$n_{01}<n_{02}<1$	
$x<\sqrt{2}\,\sigma_0$	$x>\sqrt{2}\,\sigma_0$
$\ln\left(\frac{x^2}{2\sigma_0^2}\right)<0$	$\ln\left(\frac{x^2}{2\sigma_0^2}\right)>0$
$n_{01}\ln\left(\frac{x^2}{2\sigma_0^2}\right)>n_{02}\ln\left(\frac{x^2}{2\sigma_0^2}\right)$	$n_{01}\ln\left(\frac{x^2}{2\sigma_0^2}\right)<n_{02}\ln\left(\frac{x^2}{2\sigma_0^2}\right)$
$\left(\frac{x^2}{2\sigma_0^2}\right)^{n_{01}}>\left(\frac{x^2}{2\sigma_0^2}\right)^{n_{02}}$	$\left(\frac{x^2}{2\sigma_0^2}\right)^{n_{01}}<\left(\frac{x^2}{2\sigma_0^2}\right)^{n_{02}}$
$g\,(x;\,n_{01})<g\,(x;\,n_{02})$	$g\,(x;\,n_{01})>g\,(x;\,n_{02})$

$1<n_{03}<n_{04}$	
$x<\sqrt{2}\,\sigma_0$	$x>\sqrt{2}\,\sigma_0$
$\ln\left(\frac{x^2}{2\sigma_0^2}\right)<0$	$\ln\left(\frac{x^2}{2\sigma_0^2}\right)>0$
$n_{03}\ln\left(\frac{x^2}{2\sigma_0^2}\right)>n_{04}\ln\left(\frac{x^2}{2\sigma_0^2}\right)$	$n_{03}\ln\left(\frac{x^2}{2\sigma_0^2}\right)<n_{04}\ln\left(\frac{x^2}{2\sigma_0^2}\right)$
$\left(\frac{x^2}{2\sigma_0^2}\right)^{n_{03}}>\left(\frac{x^2}{2\sigma_0^2}\right)^{n_{04}}$	$\left(\frac{x^2}{2\sigma_0^2}\right)^{n_{03}}<\left(\frac{x^2}{2\sigma_0^2}\right)^{n_{04}}$
$g\,(x;\,n_{03})<g\,(x;\,n_{04})$	$g\,(x;\,n_{03})>g\,(x;\,n_{04})$

and

(7) $$g_0(\sqrt{2}\sigma_0;\; n_{0i})=e^{-1}=0{,}368; \quad (i=1,\ldots,4).$$

The later expression (7) means that all intensity distributions $g_0(x;\, n_{0i})$ $(i=1,\ldots,4)$ have equal widths $(\sqrt{2}\sigma_0)$ at the intensity level 36,8 %. Roughly speaking, the area of the image may be devided into two parts ($y=0$, $r=x$):

(i) inner part: $0\leq x\leq\sqrt{2}\sigma_0$;

(ii) outer part: $x\geq\sqrt{2}\sigma_0$.

According to the above description and to Table 1 (bottom rows), the intensity decrease in the inner parts of the point source images is slower (with the increase of x) for larger values of n_0. In the outer parts the situa-

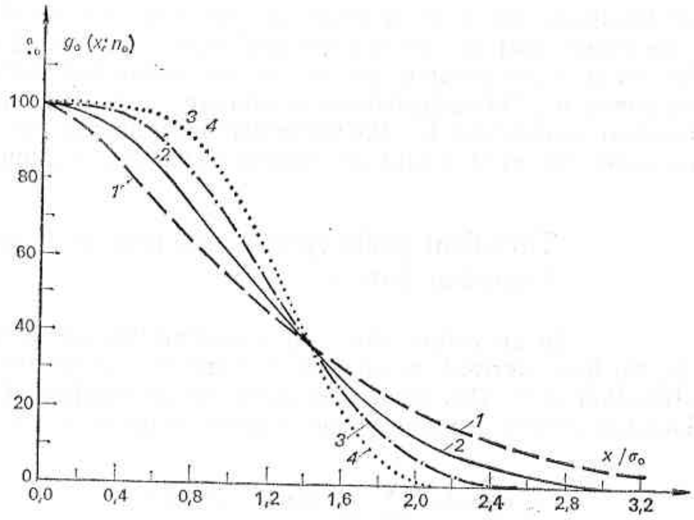

Fig. 2. Comparison between point source intensity distributions $g_0(x, n_0)$ for different values of n_0
1 — $n_0=0{,}7$; *2* — $n_0=1$ (Gaussian distribution); *3* — $n_0=1{,}4$; *4* — $n_0=2{,}0$

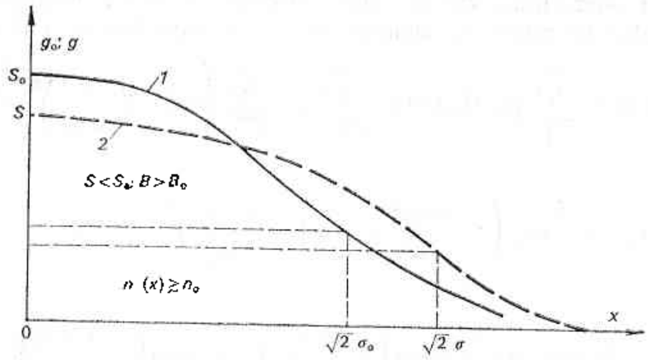

Fig. 3. Schematic comparison between the precise focused distribution $g_0(x; n_0)$ and its out-of-focus distorted ("flattened") image $g(x; n(x))$
1 — $g_0(x; n_0=\text{const})$; *2* — $g(x; n(x)\neq\text{const})$

tion is opposite: the intensities $g_0(x)$ decrease faster for larger n_0 (Fig. 2). Having in mind these considerations we are able to make a qualitative estimations about the behaviour of the power n for the out-of-focus images. If the out-of-focus distortions are not very "strong" [6], the energy flux is redistributed from the inner parts to the outer parts of the images (the total amount of the energy flux is not changed). Consequently, the intensity decreases in the inner parts and increases in the outer parts (here we do not use the normalization of the central intensities $S=1$; eq. (4)). This circumstance leads to the slower decrease of the intensity $g(x)$ in the inner parts and to its somewhat faster decrease in the outer parts in comparison with the precise focused images (Fig. 3). That is to say, the out-of-focus intensity distributions $g(x)$ are "flattened" distributions $g_0(x)$. By a rough analogy with the results in Table 1 and Fig. 2, we expect that the mean (averaged over the inner or outer parts), values of the power n are greater than the corresponding undistorted „parents" values of the power n_0. This approximate qualitative conclusion is in accordance with the numerical estimations for the particular Gaussian case ($n_0=1$) obtained in a previous paper [6], as it would be expected from the continuity reasons.

Turbulent point-spread function as a sum of Gaussian curves

In an earlier work [6], assuming that $g_0(x_0)$ is a Gaussian curve ($n_0=1$), we have derived an analytical expression about the out-of-focus intensity distribution $g(x)$. This result can easily be generalized if we consider a sum of Gaussian curves describing the intensity distribution ($y_0=0$)

$$g_0(x_0)=\sum_{i=1}^{m} p_{0i}\exp(-x_0^2/2\sigma_{0i}^2). \tag{8}$$

Here p_{0i} ($i=1,\ldots,m$) are weight coefficients which do not depend on x_0, m is the total number of the summed up Gaussian distributions with dispersions σ_{0i}^2 ($i=1,\ldots,m$), respectively. Because of linearity of the differentiation and integration operations, we are able directly (without performing intermediate calculations) to write by analogy with the equation (11) from [6]

$$(2\pi)^{-1}g(x)=\sum_{i=1}^{m} p_{0i}B_{0i}(x)+\sum_{i=1}^{m} p_{0i}\sum_{k=1}^{\infty}\left[(k!)^{-2}\left(\frac{x}{2\sigma_{0i}^2}\right)^{2k}\right]B_{ki}(x), \tag{9}$$

where

$$B_{0i}(x)=\sigma_{0i}^2\left[\exp\left(-\frac{x^2+r_1^2}{2\sigma_{0i}^2}\right)-\exp\left(-\frac{x^2+r_2^2}{2\sigma_{0i}^2}\right)\right],\quad (i=1,\ldots,m) \tag{10}$$

and

$$B_{ki}(x)=\sigma_{0i}^2\left[r_1^{2k}\exp\left(-\frac{x^2+r_1^2}{2\sigma_{0i}^2}\right)-r_2^{2k}\exp\left(-\frac{x^2+r_2^2}{2\sigma_{0i}^2}\right)\right] \tag{11}$$

$$+2k\sigma_{0i}^2B_{(k-1)i}(x),\quad (k=1,2,\ldots;\quad i=1,\ldots,m).$$

It would be noted that every coefficient p_{0i} depends, however, on σ_{0i}: $p_{0i}=p_{0i}(\sigma_{0i})$ ($i=1,\ldots,m$), by analogy with the dependence $S_0=S_0(\sigma_0)$ as in the case of a single Gaussian curve [6]. This circumstance must be taken into account if we try to perform the transition $\sigma_{0i}\to 0$ for some (or all) of the components in the sum (9). As can be seen from (10) and (11), $B_{0i}(x)$ and $B_{ki}(x)$ tend to zero when $\sigma_{0i}\to 0$ ($i=1,\ldots,m$). But the corresponding coefficients p_{0i} ($i=1,\ldots,m$) must approach infinity in such a way that the total light flux in the out-of-focus image to remain a constant, equal to $\int\int_{-\infty}^{\infty} g_0(x_0, y_0)dx_0dy_0$. Moreover, the transitions $\sigma_{0i}\to 0$ ($i=1,\ldots,m$) are physically incorrect because they do not correspond to the adopted in this paper geometrical optics approach. Consequently, we shall consider all σ_{0i} $(i=1,\ldots,m)$ as a large enough strictly positive quantities with preliminary fixed values.

Differentiating with respect to x one or two times the above three expressions (9)-(11), we shall obtain the first and second derivatives $g'(x)$ and $g''(x)$, correspondingly. We shall write in an explicite form only the final results

$$(12)\quad (2\pi)^{-1}g'(x)=\sum_{i=1}^{m}p_{0i}B'_{0i}(x)+\sum_{i=1}^{m}p_{0i}\sum_{k=1}^{\infty}\frac{2k}{(k!)^2}\left(\frac{1}{2\sigma_{0i}^2}\right)^{2k}x^{2k-1}B_{ki}(x)$$

$$+\sum_{i=1}^{m}p_{0i}\sum_{k=1}^{\infty}\frac{1}{(k!)^2}\left(\frac{x}{2\sigma_{0i}^2}\right)^{2k}B'_{ki}(x),$$

where the functions $B'_{0i}(x)$ and $B'_{ki}(x)$ are given by

$$(13)\quad B'_{0i}(x)=-x\left[\exp\left(-\frac{x^2+r_1^2}{2\sigma_{0i}^2}\right)-\exp\left(-\frac{x^2+r_2^2}{2\sigma_{0i}^2}\right)\right];\quad (i=1,\ldots,m)$$

and

$$(14)\quad B'_{ki}(x)=-x\left[r_1^{2k}\exp\left(-\frac{x^2+r_1^2}{2\sigma_{0i}^2}\right)-r_2^{2k}\exp\left(-\frac{x^2+r_2^2}{2\sigma_{0i}^2}\right)\right]$$

$$+2k\sigma_{0i}^2B'_{(k-1)i}(x),\quad (k=1,2,\ldots;\ i=1,\ldots,m).$$

The second derivative of the intensity distribution is

$$(15)\quad (2\pi)^{-1}g''(x)=\sum_{i=1}^{m}p_{0i}B''_{0i}(x)+\sum_{i=1}^{m}p_{0i}\sum_{k=1}^{\infty}\frac{2k(2k-1)}{(k!)^2}\left(\frac{1}{2\sigma_{0i}^2}\right)^{2k}x^{2k-2}B_{ki}(x)$$

$$+2\sum_{i=1}^{m}p_{0i}\sum_{k=1}^{\infty}\frac{2k}{(k!)^2}\left(\frac{1}{2\sigma_{0i}^2}\right)^{2k}x^{2k-1}B'_{ki}(x)+\sum_{i=1}^{m}p_{0i}\sum_{k=1}^{\infty}\frac{1}{(k!)^2}\left(\frac{x}{2\sigma_{0i}^2}\right)^{2k}B''_{ki}(x),$$

where

$$(16)\quad B''_{0i}(x)=\left(\frac{x^2}{\sigma_{0i}^2}-1\right)\left[\exp\left(-\frac{x^2+r_1^2}{2\sigma_{0i}^2}\right)-\exp\left(-\frac{x^2+r_2^2}{2\sigma_{0i}^2}\right)\right];\quad (i=1,\ldots,m)$$

and

$$(17)\quad B''_{ki}(x)=\left(\frac{x^2}{\sigma_{0i}^2}-1\right)\left[r_1^{2k}\exp\left(-\frac{x^2+r_1^2}{2\sigma_{0i}^2}\right)-r_2^{2k}\exp\left(-\frac{x^2+r_2^2}{2\sigma_{0i}^2}\right)\right]$$

$$+2k\sigma_{0i}^2B''_{(k-1)i}(x);\quad (k=1,2,\ldots;\ i=1,\ldots,m).$$

Taking into account that the coefficients p_{0i} $(i=1,\ldots,m)$ do not depend on x, it is possible to show that for $x=0$ the power $n(0)$ is equal to unity. Indeed, it is easy to estimate from (9), (12) and (15) that

$$(18)\quad (2\pi)^{-1}g(0)=\sum_{i=1}^{m}p_{0i}\sigma_{0i}^2\left[\exp\left(-\frac{r_1^2}{2\sigma_{0i}^2}\right)-\exp\left(-\frac{r_2^2}{2\sigma_{0i}^2}\right)\right],$$

$$(19)\quad (2\pi)^{-1}g'(0)=0$$

and

$$(2\pi)^{-1}g''(0) = \sum_{i=1}^{m} p_{0i} B''_{0i}(0) + \sum_{i=1}^{m} 2p_{0i} \left(\frac{1}{2\sigma_{0i}^2}\right)^2 B_{1i}(0) \tag{20}$$

$$= \sum_{i=1}^{m} p_{0i} \left[\exp\left(-\frac{r_2^2}{2\sigma_{0i}^2}\right) - \exp\left(-\frac{r_1^2}{2\sigma_{0i}^2}\right) + 2\left(\frac{1}{2\sigma_{0i}^2}\right)^2 B_{1i}(0)\right].$$

Obviously, the ratio $-xg'(x)/g(x)$ tends to zero when x approaches 0. To evaluate $xg''(x)/g'(x)$ for $x=0$, we must estimate the limit $g'(x)/x$ for $x \to 0$.

$$\lim_{x\to 0} [g'(x)/x] = 2\pi \lim_{x\to 0} \sum_{i=0}^{m} p_{0i} \Bigg[x^{-1}B'_{0i}(x) \tag{21}$$

$$+ \sum_{k=1}^{\infty} \frac{2k}{(k!)^2} \left(\frac{1}{2\sigma_{0i}^2}\right)^{2k} x^{2k-2} B_{ki}(x) + \sum_{k=1}^{\infty} \frac{1}{(k!)^2} \left(\frac{1}{2\sigma_{0i}^2}\right)^{2k} x^{2k-1} B'_{ki}(x) \Bigg].$$

Taking into account (13), (14) and also the expression (20), this leads to

$$\lim_{x\to 0} [g'(x)/x] = 2\pi \sum_{i=1}^{m} p_{0i} \left[\exp\left(-\frac{r_2^2}{2\sigma_{0i}^2}\right) - \exp\left(-\frac{r_1^2}{2\sigma_{0i}^2}\right) + 2\left(\frac{1}{2\sigma_{0i}^2}\right)^2 B_{1i}(0)\right] = g''(0) \tag{22}$$

Consequently

$$\lim_{x\to 0} [xg''(x)/g'(x)] = 1 \tag{23}$$

and

$$n(x=0) = 1. \tag{24}$$

This equality means that if the initial (i. e., precise focused) point source intensity distribution is a Gaussian one ($n_0=1$) or sum of Gaussian curves, then the curve $S(x)=x/[2n(x)-1]$ for x close to zero is a nearly straight line with a slope of 45° for an arbitrary value of Δf. This result is independent of the values of the coefficients p_{0i} and dispersions σ_{0i}^2, ($i=1, \ldots, m$). It would be noted that the Gaussian curves in (8) have maximal values which are not displaced from the center of the image $x_0=0$. For every single Gaussian curve term in (8) the results obtained in the previous paper [6] may be applied separately. Then for some of the terms (with small σ_{0i}) the out-of-focus distortions would be "strong", for other terms (with larger σ_{0i}) distortions would be "moderate" and, finally, for the largest σ_{0i} they (eventually) would be "slight". Having in mind that the "strong" distortions are not well described (in the case of a single Gaussian curve) by the power $n(x)$ ([6]; Fig. 3), we should use the expression (8) (or (9)) with some cautiousness if terms with small σ_{0i} are included. This remark is connected also with the difficulties which may arise with regard to the convergence of the infinite series in (9), (12) and (15) when some (or all) of the quantities σ_{0i} ($i=1, \ldots, m$) tend to zero. As mentioned earlier, description of the out-of-focus distortions by means, of the power $n(x)$ (5) is not a suitable tool in the case of "strong" distortions. In the case of a superposition of Gaussian curves (8), the later statement can be checked by assigning concrete numerical values to the number of terms m, dispersions σ_{0i}^2 $(i=1, \ldots, m)$ and weights p_{0i} $(i=1, \ldots, m)$. We shall not perform here these calculations. Qualitatively, it is evindent that if the weights p_{0i} (corresponding to small σ_{0i}, such that the out-of-focus distortions are "strong") are large, it would be expected that the power $n(x)$ is not a useful variable parameter giving the slope of the function $g(x)$. Nevertheless, the expression (9) still gives a reasonable description of the out-of-focus intensity distribution. It would be emphasized that the above conclusions about the power $n(x)$ do not concern the undistorted power n_0, if the point-spread function $g_0(r_0)$ (3) is addopted.

Conclusions

We have considered an optical system with central screening of the input aperture and the performed computations are made in the geometrical optics approach. The point-spread function of the turbulent atmosphere is approximated in two ways: (i) by a single quasi-Gaussian curve (3), and (ii) by a sum of Gaussian curves with different weights and dispersions (8). In the later case we give exact analytical expressions describing the out-of-focus intensity distribution $g(r)$. Such results may be useful when they are applied for reconstruction of out-of-focus distorted images obtained during observations through a random turbulent medium. Then $g(r)$ is simply the point-spread function of the system "turbulent atmosphere + out-of-focus telescope". We entirely neglect the distortion effects like koma, astigmatism, etc.

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Димитър Димитров

(Р е з ю м е)

Разгледани са два случая на апроксимиране на функцията на импулсния отклик за наблюдения през турбулентна среда: 1) единична квазигаусова крива $g_0(r_0) \sim \exp(-r_0^{2n_0}/B_0)$, където B_0 е константа, определяща размера на изображението на точковия източник, а за степенния показател n_0 е прието, че има постоянна стойност; 2) сума от гаусови криви $g_0(r_0) = \sum_{i=1}^{m} p_{0i} \exp(-r_0^2/2\sigma_{0i}^2)$ с различни тегла p_{0i} и дисперсии σ_{0i}^2. Ако измерванията на осветеността не са извършени във фокалната равнина на телескопа, извънфокалното разпределение на осветеността $g(r)$ също може да бъде апроксимирано с квазигаусова крива, но степенният показател n ще зависи от r. В случая 1) е дадено качествено описание на поведението на n и в случая 2) е получено точното аналитично представяне за разпределението на осветеността $g(r)$. В последния случай е показано и че $n(0)=1$ за произволни p_{0i}, σ_{0i} и извънфокално отместване Δf. Всички оценки са извършени в приближението на геометричната оптика.